Technical Notes

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System Identification by the Continuation Method

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Introduction

THIS Note is concerned with parameter identification of a system modeled by nonlinear differential equations. Discrete noisy measurements are made on the response of the system. It is desired to estimate the unknown parameters in the system model by minimizing a mean square cost function. Bellman[1] applied the quasilinearization method to solve this problem. Alternate methods for fitting differential equations to experimental data are described in Refs. 2 and 3. Convergence of these numerical approaches is critically dependent on a good prior estimation of the unknown parameters.[4] Continuation methods are suitable for extending the domain of convergence of a given iterative method.[5] The basic idea of the continuation methods is quite old[6] and rather simple: instead of the given problem, one solves a similar but simpler problem. Then this solution is "continued" until the original problem is reached. Continuation methods are of two kinds: continuous or iterative.[5]

Wasserstrom[7] presented a continuation method of the continuous type for identifying system parameters. The number of measurements was assumed to be equal to the number of unknown parameters and convergence to the solution was achieved by solving a nonlinear multipoint boundary-value problem.

The objective of this Note is to apply a continuation method of the iterative type for enlarging the domain of convergence of the quasilinearization method. This work extends a previous analysis by the authors of the problem of identifying a system modeled by a nonlinear equation.[8]

Statement of the Problem

The approach is introduced by considering the set of equations

$$\dot{Y}=F(Y,x,\Gamma) \tag{1a}$$

$$Y(0)=C \tag{1b}$$

$$z=g(Y,x) \tag{1c}$$

where x denotes the independent variable, $Y=(y_1,...y_n)^T$ is an $(n\times1)$ state vector, $\Gamma=(\gamma_1,...,\gamma_i)^T$ is an $(i\times1)$ vector of unknown coefficients, $C=(c_1,...,c_n)^T$ is an $(n\times1)$ vector of initial conditions whose ℓ first components are unknown, $F=(f_1,...,f_n)^T$ is an $(n\times1)$ vector, $f_j=f_j(y_1,...,y_n,\ x,\ \gamma_1,...,\gamma_i)$ $j=1,...,n$, z is the scalar response.

Define β the $(k\times1)$ vector of unknown parameters by

$$\beta=(\beta_1,...,\beta_k)^T \quad k=\ell+i$$

$$(\beta_1,...,\beta_k)=(c_1,...,c_\ell,\ \gamma_1,...,\gamma_i) \tag{2}$$

If f_i and g are well-behaved functions, the state vector Y and the response z solutions of Eqs. (1) are continuous and differentiable with respect to x and β. In the sequel, they are denoted

$$Y=Y(x,\beta)$$

$$z=z(x,\beta) \tag{3}$$

Noisy measurements b_m are made on z at stations x_m, $m=1,2,...,N$, where $N>n$. The estimation problem is to find the parameter vector β^j which minimizes the cost function

$$J(\beta)=\sum_{m=1}^{N}[b_m-z(x_m,\beta)]^2 \tag{4}$$

β^j is called the least squares estimate of the unknown parameter vector β of the system defined in Eqs. (1).

Formulation of the Continuation Method

Using the principle of the continuation method, the original least squares problem Eqs. (1-4) is imbedded into a one parameter family of similar problems. This family is defined as follows. Choose an initial approximation β° for β^j and define a new cost function

$$S(\beta,t)=tJ(\beta)+(1-t)\sum_{m=1}^{N}[t\,b_m+(1-t)z(x_m,\beta^\circ)-z(x_m,\beta)]^2 \tag{5}$$

where t is a parameter varying in the interval $0\leq t\leq 1$. Clearly, the function $S(\beta,t)$ is always positive. Consider the problem of finding the minimizing vector $\beta^s(t)$ of the function $S(\beta,t)$. This problem will be denoted by $\pi(t)$. Note that

$$\text{a) at}\quad t=0,\ S(\beta^\circ,0)=0 \tag{6}$$

so that the solution of the problem $\pi(0)$ is $\beta^s(0)=\beta^\circ$

$$\text{b) at}\quad t=1,\ S(\beta,1)=J(\beta) \tag{7}$$

so that the problem $\pi(1)$ coincides with the initial minimum problem and its solution is:

$$\beta^s(1)=\beta^j$$

To apply the continuation method in its discrete version, the interval $0\leq t\leq 1$ is subdivided into M equal parts so that $t_i=i/M$, $i=1,2,...,M$. Each problem $\pi(t_i)$ will be solved by an iterative minimization method, using the solution $\beta^s(t_{i-1})$ of the problem $\pi(t_{i-1})$ as a starting approximation to solve the problem $\pi(t_i)$. If $t_i-t_{i-1}=1/M$ is small enough, then hopefully $\beta^s(t_{i-1})$ belongs to the region of convergence of

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the minimization method for the problem $\pi(t_i)$, so that convergence will occur.[5] In particular, $\beta^s(0)=\beta^\circ$ is taken as the first approximation to solve the problem $\pi(t_1)$. An efficient numerical method to minimize the cost function $S(\beta,t)$ is the quasilinearization method described in the next section.

The Quasilinearization Method

Define the notations

$$\beta^s(t_i)=\beta^i \qquad i=1,2,\ldots,M \tag{8}$$

$$[\partial z(x,\beta)/\partial\beta]=H(x,\beta) \tag{9}$$

where $H(x,\beta)$ is the $(k\times 1)$ gradient vector of z with respect to β. $H(x,\beta)$ satisfies the following equations[6]

$$\dot{W}=\frac{\partial F}{\partial Y}W+\frac{\partial F}{\partial\beta}$$

$$W(o)=\begin{bmatrix} I & 0_1 \\ 0_2 & 0_3\end{bmatrix}$$

$$H=(\partial g/\partial Y)W \tag{10}$$

where W is the Jacobian matrix of Y with respect to β, I is the $(\ell\times\ell)$ identity matrix, 0_1 is a $(\ell\times i)$ null matrix, 0_2 is a $[(n-\ell)\times\ell]$ null matrix, 0_3 is a $[(n-\ell)\times i]$ null matrix, and the partial derivatives $\partial g/\partial Y$, $\partial F/\partial Y$, $\partial F/\partial\beta$ are evaluated at the solution $Y(x,\beta)$ of Eqs. (1). For a given vector β, the function $z(x,\beta)$ and its gradient $H(x,\beta)$ can be determined by integrating simultaneously Eqs. (1) and (10) using a numerical technique such as the predicator-corrector method.

Assume that β^{i-1} is close to β^i and set

$$\beta^i\approx\beta^{i-1}+\Delta\beta \tag{11}$$

where $\Delta\beta$ is a vector of small corrections. Linearize the calculated response z with respect to β^{i-1}

$$z(x,\beta)\approx z(x,\beta^{i-1})+H(x,\beta^{i-1})\Delta\beta \tag{12}$$

Substituting Eq. (12) into the expression of $S(\beta,t)$ at $t=t_i$ leads to the function

$$SS(\Delta\beta,t_i)=t_i\sum_{m=1}^{N}[b_m-z(x_m,\beta^{i-1})-H(x,\beta^{i-1})\Delta\beta]^2$$
$$+(1-t_i)\sum_{m=1}^{N}[t_i\,b_m+(1-t_i)z(x_m,\beta^\circ)$$
$$-z(x_m,\beta^{i-1})-H(x_m,\beta^{i-1})\Delta\beta]^2 \tag{13}$$

$SS(\Delta\beta,t_i)$ is a quadratic form in $\Delta\beta$. Solving for the value of $\Delta\beta$ which minimizes $SS(\Delta\beta,t_i)$ yields

$$\Delta\beta=A^{-1}B \tag{14}$$

where the $(k\times k)$ matrix A and the $(k\times 1)$ vector B are given by:

$$A=\sum_{m=1}^{N}H^T(x_m,\beta^{i-1})H(x_m,\beta^{i-1})$$
$$B=\sum_{m=1}^{N}[t_i(2-t_i)b_m+(1-t_i)^2z(x_m,\beta^\circ)$$
$$-z(x_m,\beta^{i-1})]H^T(x_m,\beta^{i-1}) \tag{15}$$

A new approximation of β^i is obtained by adding $\Delta\beta$, Eq. (14), to β^{i-1}. To compute β^i the relation (14) is applied iteratively until a convergence criterion, based for example on the variation of the function $S(\beta,t_i)$ is satisfied.

Example

$$\dot{y}=10-\gamma y^2 \tag{16a}$$

$$y(0)=c \tag{16b}$$

$$z=y \tag{16c}$$

Equations (16) describe the speed variation of a free falling object in the atmosphere. $\beta=(c,\gamma)$ is the unknown parameter vector. For testing the present approach, observations on z were generated by digital simulation of Eqs. (16) for the following value of the parameter vector β:

$$\beta=(0,\ 0.02)$$

The data abscissas were fixed equal to $x_m=0.05\ (m-1)$, $m=1,2,\ldots,30$. A zero mean Gaussian noise, with a standard deviation $\sigma=0.33$ was added to the measurements.

Figure 1 presents the domains of convergence of the continuation method and of the quasilinearization method. These domains are limited for physical reasons to the region $c>0$, $\gamma>0$. The increase in the region of convergence of the quasilinearization method, due to the application of the continuation algorithm, is clearly shown in Fig. 1.

In the application of the proposed method, the t-interval [0,1] has been subdivided into ten equal parts. Denote the minimizing vector of the function $S(\beta,t)$ (Eq. 5) by $\beta^s(t)=(c^s(t),\ \gamma^s(t))$. The t-evolutions of $c^s(t)$, $\gamma^s(t)$, using the starting vector $\beta^\circ=[20.0,\ 5.0]$ are shown in Fig. 2. The following values were obtained at $t=1$, $c^s(1)=-0.07$, $\gamma^s(1)=0.019$. About 50 iterations were needed for converging to the solution. The computation time for that case was 5 sec. on the IBM 370/165.

Conclusions

A new approach for obtaining the least squares estimates of unknown parameters of nonlinear differential models has been described. The solution uses a continuation method of the iterative type. The basic idea is to define a new cost function depending on a parameter so that the original least squares estimation problem is imbedded into a family of auxiliary minimum problems. The inital parameter guess is brought into the region of convergence of the existing parameter estimation quasilinearization method through the resolution of a finite sequence of the auxiliary minimum

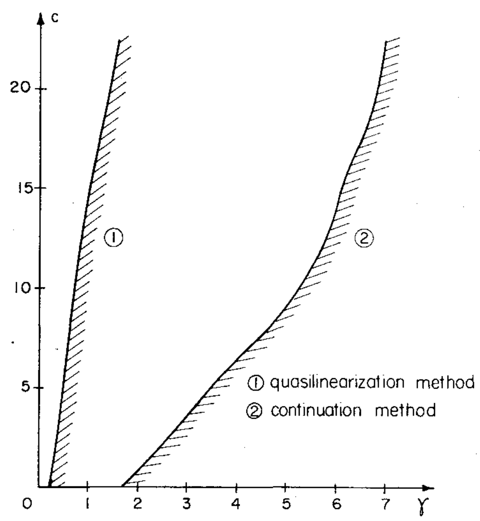

Fig. 1 Domains of convergence of the quasilinearization and continuation methods.

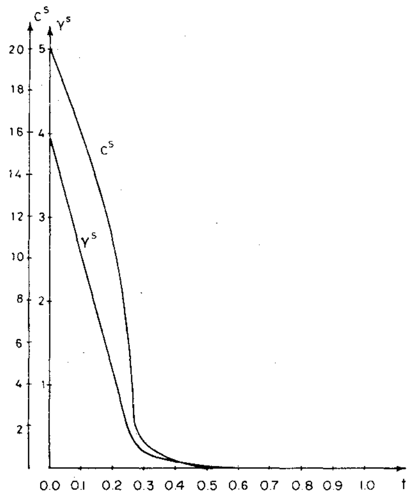

Fig. 2 *t*-evolution of the parameters c^s and γ^s.

problems. Each one of the minimum problems is solved by a quasilinearization procedure. The advantage of the proposed method is that it provides a substantial increase in the region of convergence of the quasilinearization method. The numerical experiment which is presented demonstrate clearly this useful property.

The necessary price for the observed improvement in convergence properties is a rather large computation time. More work is needed for comparing the computation times of the present method and other estimation methods having a large domain of convergence.

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Electromagnetic Fields Generated by Turbulent Air Flow and Shock Waves

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Introduction

THIS research program is a continuation of studies of electromagnetic fields radiated from regions of high shear observed outside of a glass shock tube and in cold oxygen gas flow,[1] and to a limited extent in von Karman vortices.[2] This research originated in a study of aerodynamic drag reduction.[3] The purpose of this paper is to establish the fact that electromagnetic fields are radiated from regions of high rates of change of shear in turbulent flow and shock waves.

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The electromagnetic field associated with turbulent air flow may be the result of the dynamic motion of H_2O and CO_2 polar molecules and N_2, O_2 and other molecules given induced dipole moments by polarization. The electronic and magnetic probes used, respond only to AC signals, and are therefore sensitive to only the dynamic motion of electric charges possibly associated with these dipoles.

When these natural and induced dipoles are subjected to a polarizing field which is generated by their motion, they are rotated until their polar axis has a direction opposite to the direction of the field. Tests indicate the dynamic motion of these groups of aligned molecules, and any electric charges, produce the signals picked up by the electronic and magnetic probes.

Tests Instruments and Equipment

Two types of probes were used (see Fig. 1). Both of them contain subminiature amplifiers and associate circuits in their housing. The electronic probe is used to pick up the electric vector $\boldsymbol{E}$ in the induction field. It was found the magnitude of this vector was so low that it would only induce low level electric charges in the sensing electrode of the pickup. Accordingly, the electrode was connected to an electrometer type amplifier which was designed to measure electric charges. This amplifier has an input impedance of 1×10^{13} ohms and a sub-picoampere bias current. With the proper design of the feedback circuit and termination resistance, it is possible to obtain a linearity of ± 1 db from 25 Hz to 1.5 MHz. The sensing electrode exposed to the electromagnetic field is 3 mm diam and 40 mm long.

The magnetic probe shown in Fig. 1 for picking up the magnetic vector $\boldsymbol{B}$ of the electromagnetic field in the induction zone, has a sensing element in the tip which is 8 mm diam and 9 mm long. This small sensing element will give a higher resolution to local variations in the magnetic field than the electronic probe. The subminiature amplifier used in the magnetic probe has a high gain with an input resistence of about 50 megohms.

The subsonic tests were conducted in an open return wind tunnel having a test section diam of 298 mm. A vortex generator[4] consisting of a plastic strip 80 mm high by 10 mm thick was mounted horizontally across the center of the test section perpendicular to the direction of the air flow. The top of the vortex generator was machined with a 30° angle between the sloping forward face and the vertical downstream face of the plastic strip. All of the subsonic tests were conducted at an air speed of 17.9 m/s, (40.0 mph).

The supersonic tests were conducted in the flow downstream of a small expansion-type nozzle machined in the center of a nozzle plate. This produced an underexpanded

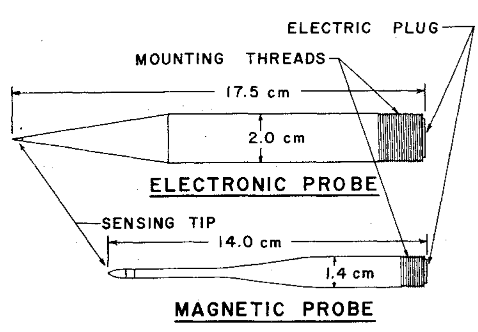

Fig. 1 Supersonic probes.